



A Multidisciplinary Indexed International Research Journal



ISSN : 2320-3714  
Volume : 4



ADHYAYAN  
INTERNATIONAL  
RESEARCH  
ORGANISATION



## FORCES IN THREE DIMENSIONS

By Mr. MukeshYadav  
Asstt. Prof. of Mathematics

**Declaration of Author:** I hereby declare that the content of this research paper has been truly made by me including the title of the research paper/research article, and no serial sequence of any sentence has been copied through internet or any other source except references or some unavoidable essential or technical terms. In case of finding any patent or copy right content of any source or other author in my paper/article, I shall always be responsible for further clarification or any legal issues. For sole right content of different author or different source, which was unintentionally or intentionally used in this research paper shall immediately be removed from this journal and I shall be accountable for any further legal issues, and there will be no responsibility of Journal in any matter. If anyone has some issue related to the content of this research paper's copied or plagiarism content he/she may contact on my above mentioned email ID.

### ABSTRACT

A rigid body is composed of many particles, the distance between the two particles remains constant, no matter what forces act on it. No perfectly rigid body exists in nature. Bodies change their sizes and shapes under the stress of great forces. When a rigid body is being acted upon by forces in different directions in space, we say that body is under the action of forces in the three dimensions.

If X, Y, Z be the components of a force R along x, y and z-axis respectively then

$$R^2 = X^2 + Y^2 + Z^2.$$

Direction cosine of line of action of R are  $\left\langle \frac{X}{R}, \frac{Y}{R}, \frac{Z}{R} \right\rangle$

Conversely if direction cosines of line of action of force R are  $\langle l, m, n \rangle$  then the components of force R along the axes are  $lR, mR, nR$  respectively. Since a couple can be specified by a straight line drawn in a certain direction, therefore it is a vector quantity. Hence couples can be resolved or compounded by the parallelogram law in the same way as the forces. If there are three component couples about the axis of x, y and z whose moments L, M, N are represented by OA, OB and OC respectively, then they compound into a single couple whose moment then  $G^2 = L^2 + M^2 + N^2$ .

Direction cosine of line of action of G are  $\left\langle \frac{L}{G}, \frac{M}{G}, \frac{N}{G} \right\rangle$

Conversely if direction cosines of line of action of force G are  $\langle l, m, n \rangle$  then the components of G along the axes are  $lG, mG, nG$  respectively.

Any given system of forces acting at different points of a rigid body can be reduced to a single force through any arbitrary chosen point and a couple whose axis passes through that point.

Any system of forces acting on a rigid body can be reduced to a single force together with a couple axis is along the direction of the forces.

## INTRODUCTION

**Any system of forces acting on a rigid body can be reduced in general to a force acting at an arbitrary chosen point of the body and a couple.**

Take the arbitrary chosen point O as the origin and the three mutually perpendicular lines OX, OY, OZ through O as axes of co-ordinates.

Let one of the forces P of the system act at A ( $x_1, y_1, z_1$ ) whose components parallel to the axes are  $X_1, Y_1, Z_1$ .

From A draw AM perpendicular on the plane XOY and from M draw MN perpendicular to OX.

$$\therefore ON = x_1, NM = y_1 \text{ and } MA = z_1$$

At N and O introduce two equal and opposite forces each equal and parallel to  $Z_1$ . These, being in equilibrium, do not affect the resultant of the system.

Now the forces  $Z_1$  along NF and  $Z_1$  along OZ' form a couple of moment  $-x_1Z_1$  in a plane perpendicular to the axis of y about OY. Again forces  $Z_1$  along NE and MG form a couple of moment  $y_1Z_1$  about OX.

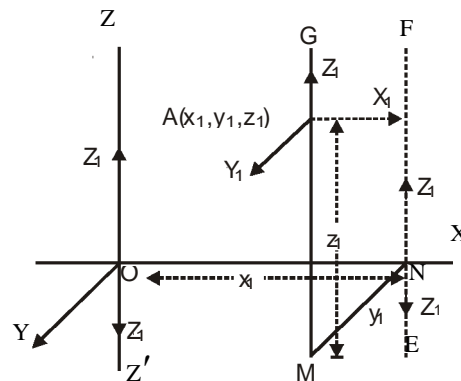
Similarly, the component  $X_1$  of the force at A( $x_1, y_1, z_1$ ) is equivalent to a single force  $X_1$  at O along OX, a couple of moments  $z_1X_1$  about OY and a couple of moment  $-y_1X_1$  about OZ.

And the component  $Y_1$  of the force at A ( $x_1, y_1, z_1$ ) is equivalent to force  $Y_1$  along OY, a couple of moment  $x_1 Y_1$  about OZ and a couple of moment  $-z_1 Y_1$  about OX.

Combining the above three results, the components  $X_1, Y_1, Z_1$  of the force at A( $x_1, y_1, z_1$ ) are together equivalent to

Therefore, we find that the force  $Z_1$  at A( $x_1, y_1, z_1$ ) is equivalent to

(i) an equal and parallel force  $Z_1$  along the axis of z



(ii) a couple of moment  $y_1Z_1$  about x-axis

(iii) a couple of moment  $-x_1Z_1$  about y-axis.

the forces  $X_1, Y_1, Z_1$  along OX, OY and OZ respectively and

a couple of moment  $(y_1Z_1 - z_1Y_1)$  about OX

a couple of moment  $(z_1X_1 - x_1Z_1)$  about OY

a couple of moment  $(x_1Y_1 - y_1X_1)$  about OZ.

Similarly the force acting at another point  $A_1(x_2, y_2, z_2)$  with components  $X_2, Y_2, Z_2$  can be replaced by forces acting along OX, OY and OZ and couples about these lines as axes.

Hence all the forces of the given system can be treated in this way so that the whole system of the forces is equivalent to

a force along  $OX = X_1 + X_2 + \dots$

a force along  $OY = Y_1 + Y_2 + \dots$

a force along  $OZ = Z_1 + Z_2 + \dots$

a couple of moment

$$\Sigma (y_1 Z_1 - z_1 Y_1) =$$

L about OX

a couple of moment

$$\Sigma (z_1 X_1 - x_1 Z_1) =$$

M about OY

anda couple of

moment  $\Sigma (x_1 Y_1 -$

$y_1 X_1) = N$  about

OZ.

The above three forces X, Y, Z are equivalent to a single force R acting through O, where  $R^2 = X^2 + Y^2 + Z^2$ .

Direction cosine of line of action of R are

$$\frac{X}{R}, \frac{Y}{R}, \frac{Z}{R}$$

Also the above couples of moments L, M, N are together equivalent to a single couple of moment G, whose axis passes through O such that  $G^2 = L^2 + M^2 + N^2$ .

Direction cosine of line of action of G are

$$\frac{L}{G}, \frac{M}{G}, \frac{N}{G}$$

Hence the system of forces acting on a rigid body has been reduced to a single force R acting through an arbitrary chosen point O and a couple G whose axis passes through O.

**Also any system of forces acting on a rigid body can be reduced to a single force together with a couple whose**

**axis lies along the direction of the force.**

**Def<sup>n</sup> Wrench:** A single force R, together with a couple G, a system (R, G) whose axis coincides with the line of action of R is said to form a wrench.

**Def<sup>n</sup> Poinso's central axis:** The axis of single couple and the line of action of R to which the system of force is reduced, is called the Poinso's central axis or simply central axis. Central axis for a system of forces is unique.

### References

- H. Dadourian, Analytical Mechanics for Students of Physics and Engineering, Van Nostrand Co., Boston, MA 1913
1. R. M. Murray, Z. Li, and S. Sastry, A Mathematical Introduction to Robotic Manipulation, CRC Press, 1994
2. R. S. Ball, The Theory of Screws: A study in the dynamics of a rigid body, Hodges, Foster & Co., 1876
3. J. M. McCarthy and G. S. Soh, Geometric Design of Linkages. 2nd Edition, Springer 2010
4. Dynamics, Theory and Applications by T.R. Kane and D.A. Levinson, 1985, pp. 90-99
5. Physics for Engineering by Hendricks, Subramony, and Van Blerk, page 148
6. Engineering Mechanics: Equilibrium, by C. Hartsuijker, J. W. Welleman, page 6
7. Dimentberg, F. M. (1965) The Screw Calculus and Its Applications in Mechanics, Foreign Technology Division translation FTD-HT-23-1632-67

8. Yang, A.T. (1974) "Calculus of Screws" in Basic Questions of Design Theory, William R. Spillers (ed.), Elsevier, pp. 266–281.